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OBSERVATIONS OF VERTICAL HUMIDITY DISTRIBUTION ABOVE THE OCEAN SURFACE AND THEIR RELATION TO EVAPORATION

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INTRODUCTION

In order to obtain information on the effect of eddy viscosity and eddy diffusion at the boundary between sea and atmosphere, simultaneous measurements of humidity at two, three or four levels between 1 and 38 meters above the sea surface were made from *Atlantis* during its cruises off the east coast of the United States during the summer of 1935. The 340 series are published in the form of averages for 115 ten-minute intervals.

It is now generally accepted that the wind speed in the lowest dekameters of the atmosphere varies as the logarithm of height, provided the lapse rate is not too far from the adiabatic (Lettau, 1939, p. 72 etc.). This is valid within the layer where the normal shearing stress[1] may be considered constant with elevation and equal to the *surface resistance*, τ_0. Accordingly the eddy viscosity coefficient must increase linearly with elevation.

In formulating these relationships it is common to introduce the *friction velocity*, defined as

$$(1) \qquad W_* = \sqrt{\frac{\tau_0}{\rho}},$$

where ρ is the density of the fluid (air). The wind speed is further proportional to this friction velocity, and is usually and most conveniently expressed in terms of the height z above the surface by

$$(2) \qquad \frac{W}{W_*} = \frac{1}{k_0} \ln \frac{z + z_0}{z_0} \cdot$$

k_0 is the *universal turbulence constant* ($k_0 = 0.4$). The *roughness parameter* z_0 depends on the nature of the surface; for a fixed, solid surface it is independent of wind speed although, when the surface has more than a single roughness scale, it may increase with elevation (Rossby, 1936a, p. 11). The roughness parameter is a small quantity, so in the sum $z+z_0$ it is negligible except very near the surface. The accompanying eddy viscosity must be

$$(3) \qquad \frac{\eta}{\rho} = k_0 W_* (z + z_0) = k_0^2 (z + z_0) \frac{W_a}{\ln \dfrac{a + z_0}{z_0}},$$

where a is a fixed height (anemometer level).

In particular Rossby (1936a) has shown that the few existing observations of wind variation with height over the sea surface follow a logarithmic distribution. For certain conditions, namely when (a) the sea surface acts as a *rough* boundary and (b) the waves are in equilibrium with the wind,[2] the roughness parameter is apparently independent of wind speed and is approximately 0.6 cm. When (b) is not fulfilled, the roughness

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[1] That is, the shearing stress across horizontal surfaces and due to vertical shear and vertical (eddy) viscosity (Rossby, 1936b, p 6).

[2] By condition (b) is meant that the length of fetch, the depth of water, and the time during which the wind has been blowing steadily are all sufficiently great so that they have no effect on the wave form.

parameter may be much larger (20 cm). From his comparison with other determinations of the surface stress, the proportionality factor in (2) is at least roughly substantiated.

Under other conditions, namely with light winds when the sea surface may act as a *smooth*[3] boundary, Rossby concludes that the wind stress is transmitted to the water through a thin laminar boundary layer. In this case there is "surface slip" so the wind does not follow the simple expression (2), but seems to be given as a function only of the friction velocity and of the logarithm of height by a universal law established by von Kármán (1934) (see below, eqs. 7 and 10).

The description above applies for a steady wind uniform over a large water surface. Under such conditions, and if the air is not saturated with respect to the water surface, there is a steady evaporation and the upward transport of water vapor by turbulence is constant with elevation. It will be shown on page 27 that eddy diffusivity[4] may in most cases be assumed identical in magnitude with eddy viscosity. Hence vapor pressure should decrease upward as the logarithm of height. If this is found to be the case, it further substantiates the linear increase of eddy viscosity and logarithmic increase of wind. This substantiation is especially valuable, because measurements of humidity over the ocean are probably more reliable than those of wind. Thus studies of the humidity gradient above the ocean surface are of considerable importance, for instance, in the determination of the surface resistance.

It is to be hoped, furthermore, that such studies will eventually make possible an independent and accurate calculation of evaporation from the ocean surface. The evaporation is given as the product of the eddy diffusivity and of the vertical rate of decrease of specific humidity. If the laws governing these two quantities can once be established, it should be possible to express them in terms of the wind speed, the water temperature, and the air temperature and humidity at a single height. The measurements of these four quantities are standard observations, so the calculation of evaporation would be entirely practicable.

Discussions of this problem, employing the modern knowledge of the eddy viscosity above the sea surface, have been made by Sverdrup (1936b, 1937) and by Millar (1937). Thornthwaite and Holzman (1939) have computed evaporation from land surfaces using measurements of humidity at two levels and the eddy diffusivity distribution according to (3).

In his second paper Sverdrup employs the humidity observations which I made in 1935. He shows that the averages for a number of groups of the measurements follow a logarithmic distribution satisfactorily. He follows the method outlined above and further assumes that, whether the surface is hydrodynamically rough or smooth, there is always next to the surface a thin laminar layer of air through which the water vapor is transported by molecular diffusion. From each of the groups he computes the thickness of the laminar layer and the rate of evaporation, both for rough surface and for smooth. Then, using these empiric values of the thickness of the laminar layer and climatic values of water temperature, specific humidity at one level and wind speed, he computes the evaporation from successive zones of the Atlantic Ocean for the two types of surface. He finds that the results for rough surface agree in general with Wüst's (1920) values of the evaporation from the same zones, if the latter are multiplied by 1.22 to bring them in

[3] It is to be emphasized that the words *rough* and *smooth* are used throughout this paper in a special sense. They do not refer to the appearance of the surface, but to its hydrodynamic character.

[4] This nomenclature follows Brunt (1939, p. 225).

accord with subsequent estimates of the average evaporation from the whole ocean.

Due to the importance of the whole problem the measurements from *Atlantis* are published here in detail, and they are furthermore subjected to a renewed discussion. The measurements under conditions of thermal stability, which Sverdrup did not use, are included in the discussion.

The method here followed is to compute the expected vertical humidity distribution from our knowledge of the vertical distribution of eddy viscosity—assumed identical with eddy diffusivity—as sketched above. It is postulated, following Millar (1937, p. 55), that von Kármán's law for a hydrodynamically smooth surface applies for a limited distance above a rough surface also. A theoretical expression for the thickness of the laminar layer in accord with von Kármán's law is utilized. Since certain details of Millar's treatment also appear unacceptable, the development is given entirely afresh here. Finally the results are compared with the observed humidity distribution.

Values of evaporation are not computed, for this appears premature until our knowledge of the whole subject can be better founded by means of further measurements. That Sverdrup finds agreement in general with another estimate of the zonal distribution of evaporation does not offer conclusive evidence that his formulation for rough surface, together with his empirical values of the thickness of the laminar layer, gives the true evaporation. Furthermore, if it is hoped that this approach to the evaporation problem will eventually yield absolute values, not just relative values, we must not check our results by comparison with other estimates of evaporation.

Wüst (1937) has published a number of series of humidity measurements at various levels above the sea surface in the Baltic. Average values for his Group II at 50, 100, 200 cm plotted against logarithm of elevation (his Abb. 5) follow closely a straight line even for the sub-groups with thermal stability, but he emphasizes that all sub-groups show relatively greater gradients between 20 and 50 cm than higher up. His observations are included in the discussion below.

METHODS OF OBSERVATION

The flow lines of the air stream are lifted and crowded in passing over a ship. Due to the crowding, wind speed measured at a given height above sea level is greater than at the same height away from the ship's influence, or it may be smaller if measured too near the deck. Only along the upper part of a tall mast in the absence of rolling can a well-exposed anemometer be expected to read approximately free air conditions; such readings alone cannot determine accurately the vertical gradient, for it is small except near the surface. Temperature measurements are not affected by the crowding, so these can be more representative than wind measurements provided the lifting of the flow lines is not too great and provided radiation and contamination from the ship are avoided. Usually the temperature gradient is smaller than that for humidity, so the latter can be measured still more accurately. Small errors make the temperature gradients presented here of little value, while the humidity gradients are not too greatly affected.

Preliminary experiments showed that it was impractical to determine gradients by using one instrument successively at different heights, since the conditions often change during such a series. Thus in order to have measurements at four levels it is essentially necessary to have four instruments.

In the present case the four instruments available were three Assmann psychrometers of different types and a hygrothermograph. It is generally disadvantageous to use dissimilar instruments because their errors are different (with psychrometers chiefly because of different ventilation speeds), but in this case each was peculiarly adapted to the position where it was used.

The hygrothermograph was used near the masthead at 38 meters (sometimes at 18 meters). This instrument was converted from an aircraft barothermograph by replacing the aneroid with a hair element. It could be read to a tenth of a degree and to a fifth of one per cent. When not in use it was kept in a moist chamber, which was partly responsible for the fact that the hair element was very reliable; during the summer its calibration changed by only about 2%. The temperature element, a closed Bourdon tube, behaved very badly due to corrosion; its readings were reliable only when a daily calibration was made. Since the instrument was left in position for long intervals, its lag was of no disadvantage. The cover of the original barothermograph was removed and the bare instrument suspended by elastic tape inside a portable shelter. This shelter was closed on the four sides with an insulating material and partly open at the bottom, the air being circulated outward through radiation baffles at the top by an electric fan. In use the shelter was suspended from a shackle running free on the forestay and could be hoisted to any height; the rubber-covered cable for the fan served as downhaul and was kept under strain to prevent the shelter from swinging. Calibrations were made under cabin or deck conditions against one of the Assmanns.

A Fuess spring-driven Assmann was used at the 8-meter level. It was held rigidly 15 cm aft of the forestay in a wooden bracket, and was read directly by an observer in the boatswain's chair suspended nearby. The exposure seemed a good one except occasionally when the observer was unable to keep entirely clear of the instrument. This psychrometer was kindly lent for the purpose by the Blue Hill Meteorological Observatory.

An electric Assmann by Casella was suspended at the ship's stem with its mouth pieces on a level with the rail and a half meter forward of it, 4 meters above the water. It was read directly by an observer in the extreme bow of the boat.

The third Assmann was a reversing one by Richter and Wiese (Böhnecke, 1933). This was suspended from the rail at the stem and tended by the same observer who read the electric psychrometer, and who also recorded all the readings. It was lowered to 2 or 1.5 meters, or alternately to 2 and 1, above the surface as the sea permitted.

By keeping the reversing psychrometer outside the rail while reading and winding it, it could safely be reversed a minute after lowering it again; thus the series of readings could be repeated every three minutes. After dipping the wet bulbs at least two minutes was usually allowed before reading. The psychrometers at times seemed to be affected by direct solar radiation, but this could not be proven or evaluated. All the psychrometer thermometers were calibrated at 2°-intervals under total immersion. Corrections in nearest tenths of a degree have been applied to all readings.

The exposures of the four instruments are shown in Figures 1 and 2. No explanation seems necessary other than that the positions were chosen with three ideals in mind: (a) a spacing such that each was roughly twice as high as the one below it, so that approximately equal differences of humidity would occur between successive pairs, (b) exposures that would give as near to free air conditions as possible, and (c) availability in rigging and reading.

Readings were made only when the wind relative to the ship was forward of the beam; at other times the air reaching the instruments was disturbed in its flow or even contaminated. The other work of the ship frequently required much maneuvering, so the measurements were often interrupted due to this necessity of head winds. The measurements were further interrupted by necessary repairs to the apparatus (including washing the reversing psychrometer after immersion by unusually large waves), and by fog or rain or fresh winds, the last causing so much salt spray that accurate readings were impossible. At other times an additional observer was not available for taking readings at 8 meters. The bow of *Atlantis* is the only part which gives a suitable exposure close to the water. Unfortunately this part experiences greater vertical motion than any other, which often made measurements close to the surface impossible. The following gives approximately the maximum wind which was accompanied by a sea permitting readings at the various levels:

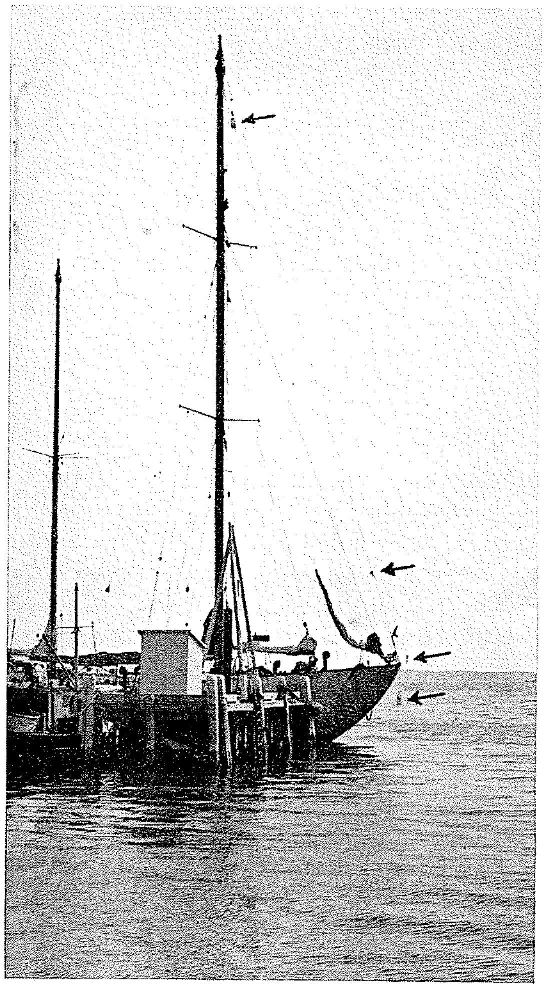

FIG. 1. Positions of four instruments when in use. From top to bottom: hygrothermograph, spring-driven Assmann (bracket only), electric Assmann, reversing Assmann.

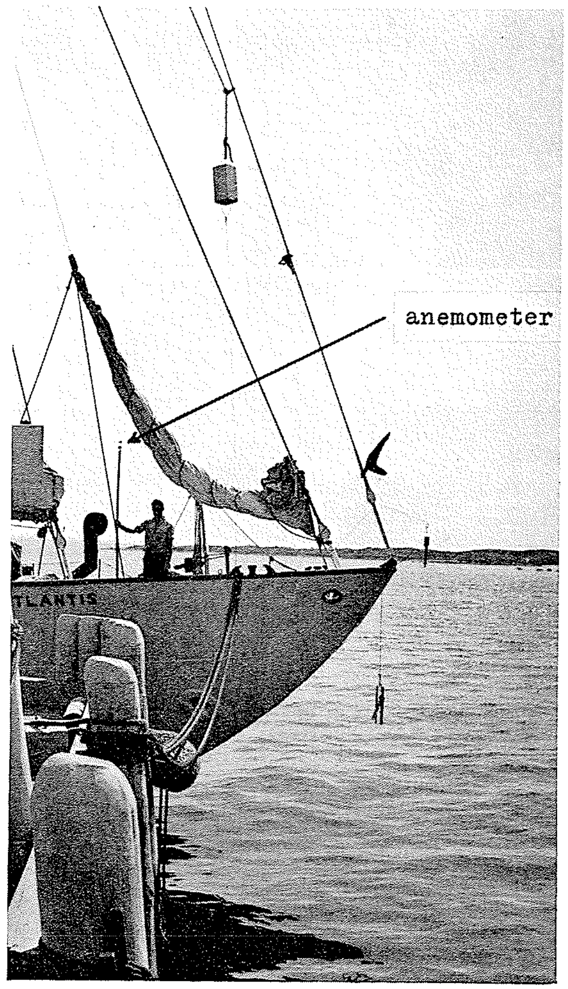

FIG. 2. Closer view of the same instruments as in Figure 1. The hygrothermograph is lowered nearly to the deck.

1 meter	2.5 m sec^{-1}
1.5	4
2	5.5
4	7.5

The wind speed was measured whenever the ship was stationary or moving slowly; when under way the Beaufort estimate was considered just as reliable as a measurement roughly corrected for the ship's motion. A small portable anemometer attached to the top of a hand pole was exposed over the windward rail 6 meters above the water, in the position shown in Figure 2. The readings have been corrected according to a careful calibration (Montgomery, 1936), as well as for the ship's headway.

During the humidity measurements the water temperature was frequently determined with a surface thermometer, always used so as to avoid any possible contamination or upwelling due to the ship.

TABULATION OF DATA

The Smithsonian Meteorological Tables (Smithsonian Institution, 1933, Table 79) were used for obtaining saturation values of vapor pressure. The percentile lowering of saturation vapor pressure by a salinity of $S°/_{oo}$ is 0.000537 S (Witting, 1908, p. 173), so saturated air over ocean water has a vapor pressure 98% of that over fresh water of the same temperature. (This constant value is correct within 0.1 mb for water below 25° with salinity between 27 and 39 $°/_{oo}$.) From this has been found the saturation vapor pressure e_s corresponding to the surface water temperature T_s.

The psychrometer readings were reduced by means of the following formula from the Smithsonian Tables:

$$e = e' - 0.000660p(T - T')(1 + 0.00115T') \tag{4}$$

Here e is the resulting vapor pressure, e' the saturation pressure corresponding to the wet bulb temperature T', T the dry bulb temperature and p the barometric pressure. p may be assumed constant at 760 mm or 1013.3 mb, so the relation used was:

$$(5) \qquad \begin{aligned} e &= e' - 0.68(T - T') \qquad 8° \leqq T' \leqq 21.5° \\ e &= e' - 0.69(T - T') \qquad 21.6° \leqq T' \leqq 34° \end{aligned}$$

Tables 1 and 2 present the data.

On the date line are given the *Atlantis* cruise number, the noon position, and the true wind direction and sea direction and disturbance (scale of 0–9) during the period covered by the

TABLE 1

TEMPERATURE (T IN °C.) AND VAPOR PRESSURE (e IN MB) AT DIFFERENT HEIGHTS FOR CRUISE 45. W_6 IS WIND SPEED 6 METERS ABOVE THE SURFACE IN M SEC^{-1}.

Time	W_6	Series	T_S	T_1	$T_{1.5}$	T_2	T_4	T_8	T_{38}		T_S-T_4	e_S	e_1	$e_{1.5}$	e_2	e_4	e_8	e_{38}		e_S-e_4	Γ	Clouds
July 21, 39°23′N. 69°52′W., wind ENE, sea ENE 2																						
1055	2.9	1	22.4		21.5		21.5		21.5	21.1	0.9	26.6		24.1		23.7		23.1	22.5	2.9	.14	very thin Stcu+Ci
1110	2.9	1	22.4		21.9		21.8		21.8	21.4	0.6	26.6		24.3		23.9		23.4	22.9	2.7	.15	
1120	(2.9)	1	22.4		22.2		22.0		21.1	21.6	0.4	26.6		24.3		24.0		23.7	22.95	2.6	.12	
1231	2.8	1	23.6		23.0		23.1		23.0	22.7	0.5	28.6		25.5		25.0		24.8	24.35	3.6	.14	3 Frcu
1325	2.8	1	23.9		23.3		23.3		23.4	22.9	0.6	29.1		25.3		24.9		24.5	23.7	4.2	.10	6 Frcu
1416–23	2.5	2	23.8	23.0		22.9	22.9		23.3	22.5	0.9	28.9	26.1		25.9	25.7		25.6	24.4	3.2	.09	2 Cu, Ci haze
1456	2.2	1	23.8			23.1	23.3	22.9	23.2	22.8	0.5	28.9			26.4	26.0	26.1	25.4	24.8	2.9	.07	
1610–17†	2.6	2	23.8	23.3		23.1	23.1	22.8	23.4	22.65	0.7	28.9	26.0		25.7	25.5	25.45	25.5	24.4	3.4	.08	slight haze only

July 23, 36°27′N. 68°07′W., wind SWxW, sea SWxW 2																						
1355	(4.6)	1	27.4		27.1		27.0		27.0		0.4	35.8		26.5		25.6		23.4		10.2-	.09	clear, Cu in distance
1438–43	(4.2)	2	27.5		27.0		27.1		27.0		0.4	36.0		27.4		26.0		25.1		10.0-	.14	
1510–14	3.9	2	27.5		27.15		27.35		27.3‡		0.15	36.0		27.6		26.7		25.6‡		9.3-	.04	Cu overhead
July 25, 35°45′N. 68°24′W., wind SW, sea SW 4																						
1112–13	7.8	2	27.4				26.9*	26.85	26.9‡	26.75‡	0.5	35.8				30.7*	29.05	27.4‡	27.15‡	5.1-	.46	2 Cu
1120–23	7.8	2	27.4				27.0*	26.95	26.8	26.65	0.4	35.8				30.2*	28.85	26.7	26.5	5.6-	.27	
July 29, 35°54′N. 68°46′W., wind WxN, sea W 2–3																						
1850–55	3.7	2	26.3			25.75	25.85	25.4	25.6		0.45	33.5			25.4	24.6	24.8	24.2		8.9-	.05	7 Cu+Ast
1905	3.4	1	26.3			25.5	25.6	25.3	25.5‡		0.7	33.5			25.9	25.1	25.3	24.7‡		8.3-	.07	
1919–24	3.0	2	26.2			25.4	25.4	25.3	25.5‡		0.8	33.3			26.0	25.6	25.45	25.1‡		7.7-	.05	
1936–46	2.5	3	26.2			25.4	25.4	25.3	25.2		0.8	33.3			25.9	25.6	25.1	24.8		7.7-	.08	6 Cu+Ast
July 30, 35°31′N. 68°46′W., wind WSW, sea SWxW 3																						
1106	6.4	1	26.1				25.3	25.0	25.55	24.7	0.8	33.2				23.3	23.05	21.9	20.8	9.9-	.11	6 Ast
1115–17	6.2	2	26.1				25.05	25.0	25.6	24.7	1.05	33.2				24.15	24.2	22.75	21.5	9.1	.13	
1253–1303†	5.0	4	26.2				25.4	25.4	25.8	25.1	0.8	33.4				25.6	25.7	25.6	24.5	7.8	−.02	2 Cu, 2 Acu
1447–50	4.0	2	26.2				26.0	25.8	26.1	25.5	0.2	33.4				25.45	25.55	25.35	24.5	8.0-	−.02	
1454–58	4.0	3	26.2				25.9	25.6	26.1	25.3	0.3	33.4				25.5	25.6	25.5	24.4	8.0-	−.02	5 Cu+Ast+Cist
1556–1602	4.3	2	26.2				25.7	25.35	26.0	25.05	0.5	33.4				25.45	25.45	25.85	24.5	8.0-	.00	5 Cu+Ast+Cist
1606–07	4.3	2	26.2				25.75	25.3	26.0	25.0	0.45	33.4				26.1	25.8	25.9	24.5	7.3-	.06	
July 31, 37°12′N. 69°48′W., wind W, sea WxS 3																						
1104–09	(4.6)	2	25.3			23.9	23.95	23.6	24.4	23.3	1.35	31.7			18.3	17.4	16.65	15.6	14.55	14.3-	.08	5 Ast+Acu
1120–28	(4.6)	3	25.3			23.9	23.9	23.6	24.4	23.3	1.4	31.7			18.3	17.5	16.6	16.1	15.0	14.2-	.09	
1242–52	(4.6)	3	25.2			24.3	24.1	23.9	24.5	23.6	1.1	31.5			20.3	19.6	18.7	17.6	16.7	11.9-	.10	6 Ci+Cicu
1256–1303	(4.7)	3	25.2			24.3	24.1	23.9	24.5	23.6	1.1	31.4			20.6	19.6	18.9	18.15	17.1	11.8-	.10	
1342–45	(5.9)	2	25.0			24.75	24.4	24.15	24.65	23.85	0.6	31.1			21.75	21.15	20.3	19.7	18.7	10.6-	.10	4 Ci+Cicu
1350–56	(5.9)	3	25.0			24.7	24.4	24.2	24.7	23.9	0.6	31.1			22.3	21.9	21.3	20.6	19.7	8.8-	.10	
1440–47	(5.9)	3	24.9			24.6	24.5	24.2	24.9	23.9	0.4	30.9			22.3	22.2	21.75	21.3	20.1	8.7-	.09	7 Ci+Cicu
1450–57	(6.0)	3	24.9			24.7	24.6	24.3	24.9	24.0	0.3	30.9			22.8	22.3	21.5	21.5	20.3	8.6-	.10	
1555–1605	7.1	4	24.8			24.75	24.65	24.5	25.05		0.15	30.7			22.45	22.4	21.4	21.35		8.3-	.04	9 Cist
1608–16	6.9	3	24.8		24.9		24.9	24.6	25.1		−0.1	30.7		21.8		20.2	19.75	20.0		10.5-	.05	
August 1, 37°21′N. 70°26′W., wind SW, sea SW 3																						
1101–08	(7.2)	6	25.0				25.8	25.8	26.95		−0.8	31.1				30.4	30.3	30.9		0.7	—	7 Cu+Acu
1454–58	6.5	4	25.0				26.15	25.7	26.8		−1.15	31.1				30.15	30.4	31.4		0.95	—	6 Cist
1601–06	6.6	6	25.0				25.9	26.0	27.1		−0.9	31.1				30.8	30.8	31.8		0.3	—	8 Cist
August 2, 37°36′N. 70°29′W., wind WSW, sea WxS 2–3																						
1603–06	(4.6)	2	24.8		25.25		25.25	25.1	25.9		−0.45	30.7		29.75		29.65	29.45	29.1		1.1	.16	6 Acu
1611–19	(4.3)	3	24.9		25.3		25.3	25.1	25.9		−0.4	30.9		29.7		29.6	29.6	29.2		1.3	.05	
1715–20	(2.6)	3	25.1		25.5		25.5	25.3	25.9		−0.4	31.1		28.6		28.3	28.2	27.6		3.0	.08	5 Cicu
1723–26	(2.6)	2	25.1		25.45		25.4	25.15	25.9		−0.3	31.3		28.75		28.5	28.35	27.75		2.8	.09	
1854–1902	(2.6)	3	25.2			25.0	24.9	24.8	25.4		0.3	31.4			28.9	28.75	28.7	28.4		2.7	.05	1 Ci
1906–12	(2.6)	2	25.2			25.05	24.95	24.8	25.3		0.25	31.4			28.9	28.7	28.6	28.4		2.7-	.08	
August 3, 40°38′N. 71°13′W., wind NE, sea NE 2																						
1218–25	(4.6)	3	23.0		21.6		21.4	21.1	21.0	20.8	1.6	27.5		18.2		17.7	17.2	16.1		9.8	.06	9 Ast
1230–34	(4.6)	2	23.0		21.6		21.4	21.1	21.0	20.8	1.6	27.5		18.0		17.5	17.0	16.1		10.0	.06	
1317–26	(4.6)	3	23.0		21.8		21.6	21.5	21.1	21.1	1.4	27.5		16.45		15.75	15.2	14.3		11.8	.06	1 Acu
1330–38	(4.6)	3	23.0		21.7		21.4	21.3	21.1	21.0	1.6	27.5		17.1		16.7	16.05	14.8		10.6	.06	
1415–24	(3.0)	3	23.2	21.7	21.6	21.7	21.5	21.4	21.2	21.1	1.7	27.9	16.2	15.9	16.0	15.75	15.6	14.5		11.7	.02	1 Ci+Acu
1430–39	(2.6)	3	23.3	21.8		21.8	21.6	21.6	21.3	21.3	1.7	28.0	14.7		14.6	14.3	14.3	13.6		13.7	.01	
1442–49	(2.6)	3	23.4	22.1		22.0	21.8	21.7	21.3	21.4	1.6	28.1	13.9		13.3	13.3	13.4	12.3		14.8	.02	

* Interpolated.

† Air samples also were obtained at this time.

‡ At 18 instead of 38 meters.

TABLE 2

TEMPERATURE (T IN °C.) AND VAPOR PRESSURE (e IN MB) AT DIFFERENT HEIGHTS FOR CRUISES 46 AND 47. W_6 IS WIND SPEED 6 METERS ABOVE THE SURFACE IN M SEC^{-1}.

Time	W_6	Series	T_S	T_1	$T_{1.5}$	T_2	T_4	T_8	T_{38}	T_S-T_4	e_S	e_1	$e_{1.5}$	e_2	e_4	e_8	e_{38}	e_S-e_4	Γ	Clouds
Cruise 46, August 7, 39°38′N. 72°27′W., wind ENE, sea ENE 3																				
1443	6.3	1	22.2		20.9		20.8		20.1	1.4	26.3		18.5		17.6		15.3	8.7	.11	10 Acu
1525	(6.6)	1	22.3			20.8	20.6		20.2	1.7	26.4			18.6	18.5		15.7	8.5	.12	9 Acu
1539–45	(6.8)	6	22.3				20.5		20.3	1.8	26.4				18.0		15.9	8.4	.11	
1631–38	(7.2)	8	22.5				20.9		20.2	1.6	26.7				17.95		16.0	7.75	.11	8 Acu
August 9, 37°40′N. 73°53′W., wind NExE, sea NExE 3																				
1913–19	4.6	2	24.9			22.8	22.55		22.45	2.35	30.9			17.8	17.35		15.75	13.55	.06	clear
1933–40	4.3	3	24.9			22.6	22.55		22.3	2.35	20.9			16.9	16.65		14.95	14.35	.03	
1944–49	4.1	3	24.9			22.7	22.6		22.3	2.3	30.9			17.3	16.8		15.2	14.1	.05	
August 10, 37°26′N. 74°29′W., wind NExE—ExS, sea ExS 1 (swell ENE 2)																				
1230	3.9	1	24.8			22.9	22.5		22.4	2.3	30.7			17.3	17.6		15.2	13.1	−.03	clear
1300–09	3.5	4	24.9		22.7	22.9	22.4		22.4	2.5	30.9		17.7	17.4	17.1		15.55	13.8	.04	
1314–23	3.4	4	24.9			22.7	22.5		22.4	2.4	30.9			17.0	16.8		15.8	14.1	.02	clear
1407–14	2.9	3	24.9		22.5		22.3		22.3	2.6	30.9		17.6		17.2		16.5	13.7	.03	
1417–26	2.9	3	24.9		22.6		22.3		22.3	2.6	30.9		16.9		16.9		15.9	14.0	.00	clear
1510–18	2.9	3	24.8		22.6		22.4		22.3	2.4	30.7		17.7		17.2		16.4	13.5	.04	
1520–29	2.9	4	24.8		22.6		22.4		22.4	2.4	30.7		17.7		17.8		16.45	12.9	−.01	clear
August 11, 38°10′N. 73°51′W., wind SSW, sea S 2																				
1548–50	6.0	3	23.8				24.5		24.2	−0.7	28.9				26.8		26.3	2.1	.11	10 Stcu
1600–06	6.0	3	23.8			24.5	24.5		24.2	−0.7	28.9			27.1	26.9		26.5	2.0	.10	
1758–1801	5.2	4	23.8				24.4		24.15	−0.6	28.9				27.4		26.95	1.4	.14	10 Stcu
August 12, 38°26′N. 73°31′W., wind ExS—SSE, sea SSE 1 (swell S 2—confused 1)																				
1100–06	2.5	2	22.8	22.65			22.45		22.85	0.35	27.15	24.65			24.45		23.8	2.7	.06	clear
1117–23	2.2	3	23.0	22.8		22.7	22.6		22.7	0.4	27.5	24.3		24.25	24.25		23.7	3.25	.01	
1253–1300	(2.3)	4	23.2	22.9		22.7	22.4		22.65	0.8	27.9	24.7		24.7	24.55		24.1	3.35	.03	clear
1304–07	2.3	2	23.3	22.7		22.7	22.45		22.5	0.85	28.0	24.6		24.4	24.55		24.0	3.45	.01	clear
1624–28	(2.3)	2	23.75		23.3		22.7		23.0	1.05	28.8		24.3		24.2		23.8	4.6	.02	3 Cu
1647–57	(2.3)	2	23.8		23.05		22.75		22.95	1.05	28.9		24.6		24.25		23.8	4.65	.08	clear, slight haze
1700–02	(2.3)	2	23.8		22.9		22.8		22.9	1.0	28.9		24.5		24.3		23.85	4.6	.04	
1837–47	(2.3)	4	23.7		23.2		23.0		23.1	0.7	28.7		24.8		24.45		24.0	4.25	.08	
August 13, 39°32′N. 72°20′W., wind SSW, sea SSW 1																				
1047	4.7	1	21.9		22.8		22.6	22.5	22.5	−0.7	25.8		24.8		24.2	24.1	24.0	1.6	.26	2 Ast, slight haze
1054–59	4.7	2	21.9		22.8		22.55	22.55	22.6	−0.65	25.8		24.6		24.15	24.3	24.15	1.65	.11	
1130–37	4.8	2	22.1		22.85		22.5	22.6	22.6	−0.4	26.1		24.85		24.5	24.35	24.2	1.6	.19	
1143–48	4.8	2	22.2		22.85		22.55	22.6	22.6	−0.35	26.2		24.55		24.4	24.35	24.2	1.8	.07	
1322	4.8	1	22.7		23.4		22.9	23.0	22.8	−0.2	27.1		25.7		25.2	25.1	24.9	1.9	.19	1 Ast, slight haze
1408–13	4.3	2	22.8		23.2		22.9	22.9	23.1	−0.1	27.2		26.1		26.1	25.7	25.5	1.1	.22	
1418–21	4.4	2	22.9		23.2		22.9	23.0	23.15	0.0	27.4		26.1		26.1	25.8	25.6	1.3	.14	
1605–08	5.3	2	22.9		23.6		23.25	23.2	23.1	−0.35	27.4		26.9		26.55	26.45	25.8	0.85	—	
1620–24	5.4	2	22.9				23.4	23.25	23.15	−0.5	27.4				26.6	26.55	25.9	0.8	—	
Cruise 47, August 20, 42°08′N. 69°58′W., wind SW—SxE, sea SxE 2																				
1016–20	3.0	3	21.0				20.3		21.1	0.7	24.4				22.5		22.0	1.9	—	3 Ci, becoming thick fog
1539–49	4.1	4	21.4	20.9		21.1	20.9		21.5	0.5	25.0	22.7		22.65	22.45		20.15	2.55	.07	7 Cist
1551–59	4.4	4	21.5	21.2		21.25	21.15		21.8	0.35	25.2	22.8		22.85	22.4		20.6	2.8	.10	
2012	3.8	1	22.0				22.0		22.3	0.0	25.9				25.3		24.5	0.6	—	5 Cist
2052–58	3.6	2	22.0		22.15		22.05		22.3	−0.05	25.9		25.45		25.3		24.5	0.6	—	3 Cist
2103–07	(3.6)	2	22.0		22.15		22.1		22.3	−0.1	25.9		25.45		25.3		24.5	0.6	—	

August 21, 41°39′N. 69°00′W., wind SxE, sea SxE 2																
1232–38	(4.6)	3	22.3	22.7	22.5		22.6	−0.2	26.3	25.3	25.1		23.8	1.2	.17	4 Ci+Cicu etc.
1242–45	(4.6)	2	22.15	22.55	22.5		22.6	−0.35	26.15	25.3	25.0		23.8	1.15	.27	
1338–44	(4.7)	3	22.2	22.6	22.4		22.7	−0.2	26.2	25.4	25.2		24.0	1.0	.20	6 Ci+Acu
1347–52	(4.9)	3	21.9	22.3	22.2		22.7	−0.3	25.9	25.2	25.0		24.1	0.9	—	
1429–36	(5.8)	3	21.5	22.0	21.9	21.9	22.4	−0.4	25.1	24.7	24.5	24.3	23.7	0.6	—	5 Ci
1439–45	(5.6)	3	21.0	21.8	21.7	21.8	22.5	−0.7	24.3	24.45	24.4	24.2	23.8	−0.1	—	
1529	(4.6)	1	20.5	20.0	20.0		22.1	0.5	23.6	23.0	22.8		23.1	0.8	—	3 Ci
August 23, 42°00′N. 69°16′W., wind NxE, sea NxE 3																
1527–35	7.8	8	21.9		19.9		19.45	2.0	25.8		18.95		17.6	6.85	.09	3 Frcu
1621–29	8.2	8	21.8		20.1		19.4	1.7	25.6		17.7		15.6	7.9	.12	clear
1811–19	6.3	8	21.25		20.1		19.5	1.15	24.75		15.0		12.7	9.75	.10	clear
1913–15	(5.4)	3	20.8		19.9		19.4	0.9	24.1		15.3		13.7	8.8	.08	clear
August 25, 40°51′N. 68°44′W., wind NW—WxN, sea NW 3																
1310–15	6.8	6	14.2		15.0		15.2	−0.8	15.9		13.65		13.1	2.25	.11	9 Cist
1428–32	5.3	5	14.3		14.9		15.2	−0.6	16.0		14.1		13.3	1.9	.20	10 Cist
1535–40	5.8	6	14.35		15.1		15.3	−0.75	16.05		14.2		13.6	1.85	.14	10 Cist
1623–33	(6.9)	7	14.2		15.1	15.1	15.2	−0.9	15.9		14.5	14.5	13.9	1.4	.19	10 Cist
1748–57	(8.4)	7	18.2		17.0	16.9	16.3	1.2	20.5		14.1	13.7	13.2	6.4	.06	10 Acu
1843–49	(7.1)	4	18.4		17.9		17.5	0.5	20.75		14.05		13.2	6.7	.06	9 Cicu+Cist
1853–54	(6.9)	2	18.7		18.1		17.5	0.6	21.2		14.0		13.0	7.2	.06	
1924–30	(6.0)	6	19.3		19.05		18.3	0.25	21.95		13.1		11.6	8.85	.08	8 Acu+Cist
August 26, 40°45′N. 70°53′W., wind WxN, sea WxN 3																
0725–31	(4.6)	6	23.0		21.3		20.9	1.7	27.45		16.5		15.9	10.95	—	9 Acu
0839–48	(4.6)	3	22.1	21.4	21.2	21.1	20.8	0.9	26.2	16.5	15.8	14.9	14.6	10.4	.09	8 Acu
0852–59	(4.6)	3	21.4	21.2	21.1	21.1	20.8	0.3	25.0	17.0	16.6	16.1	14.8	8.4	.06	
0933–39	(4.6)	3	21.1	21.2	21.3	21.1	20.9	−0.2	24.5	17.3	16.6	16.4	15.1	7.9	.07	5 Acu+Ast
0943–49	(4.8)	3	21.1	21.3	21.3	21.1	21.1	−0.2	24.5	18.4	17.8	17.6	16.0	6.7	.07	
0952–55	(4.9)	2	21.1	21.3	21.3	21.2	21.1	−0.2	24.6	18.0	17.1	17.0	15.7	7.5	.08	

day's measurements.

The first column is standard time of the 60th meridian. The wind speed, second column, is enclosed in parentheses if an anemometer reading was not made within one hour of the observation. The third column gives the number of series of humidity measurements made within the time interval of the first column (the interval never exceeds 10 minutes); if readings are shown at both 1 and 2 meters, the number of readings at each of these is less than the number of series, since the same psychrometer was used at both levels. In the following columns of temperature and vapor pressure, the subscript s refers to the sea surface and the numerical subscripts to heights in meters above the surface. Then follows Γ, and in the last column the cloud amount in tenths of the sky covered.

The quantity Γ is introduced to represent the vertical gradient of vapor pressure. The air in immediate contact with the water is assumed to be saturated with respect to the sea water. Since the vertical distribution of diffusivity is unaffected by humidity, it follows that under steady conditions the vapor pressure gradient is proportional to $e_s - e_b$, where b is any standard level for vapor pressure. Accordingly the effect of $e_s - e_b$ may be immediately eliminated by the following definition, which furthermore assumes a logarithmic distribution:

$$\Gamma \equiv - \frac{1}{e_s - e_b} \frac{de}{d \ln z} \tag{6}$$

For the present measurements the standard level is chosen to be 4 meters. In cases where the wind is stronger than 5 m sec^{-1}, $de/d \ln z$ is computed from the humidity measurements at the two extreme levels. Since the turbulent boundary layer may at times be of limited height (see p. 20), the gradient is computed from the lowest level and from the highest below 38 meters in cases where the wind does not exceed 5 m sec^{-1}. No value is given for Γ when $e_s - e_4 < 1$ mb, because the error is large when the air is nearly saturated with respect to the water.

As mentioned in the last section, the temperature readings of the hygrothermograph are unreliable for days when no calibration was carried out (July 21, 25, 30, 31 and August 1–3). Accordingly for such days (except the last two lines for July 31 and for August 1 and 2, when $T_{38} > T_s$ indicates at least possible thermal stability) the vapor pressure has been computed from the observed relative humidity and from the temperature extrapolated from the temperatures at 4 and 8 meters according to a lapse rate of 1° per 100 meters. These days occur only during Cruise 45, which is therefore given separately in Table 1. Here the columns for T_{38} and e_{38} are double, the first sub-column giving the observed value and the second the computed one. The computed vapor pressures are used in determining Γ on July 25, 30, 31.

DISTRIBUTION OF EDDY VISCOSITY ABOVE THE SEA SURFACE

It is convenient in discussing vertical mixing processes to divide the atmospheric layer of frictional influence into three mutually exclusive parts. The *outer layer of frictional influence* constitutes about 90% of the thickness of the whole layer and is characterized by a turning of the wind according to a modified Ekman spiral. Beneath this is the *turbulent boundary layer*, constituting about 10% of the whole thickness. It is characterized by the relationships (2) and (3), that is by a logarithmic wind distribution. Immediately in contact with the sea surface, or with any object over which the wind blows, there is apparently a *laminar boundary layer* of the order of magnitude of one millimeter, in which turbulence is absent. Only the two lower layers are of importance in the present study.

Within the turbulent boundary layer the distribution of eddy viscosity is given by (3) in terms of the wind speed and of the roughness parameter. In the laminar boundary layer only molecular viscosity is present. To specify the distribution of viscosity completely, then, it is necessary to know the thickness of the laminar layer and the roughness parameter.

There are two means by which the normal shearing stress, which is nearly constant throughout the turbulent boundary layer,[5] can be transmitted to the bounding surface. It may be transmitted as a normal stress through the laminar layer, as over smooth surfaces like the sea in light winds (and thermal stability). Or the process may occur chiefly by means of local horizontal pressure gradients against the sloping sides of obstacles, as over hydrodynamically rough surfaces like the ground or the sea in strong winds (and thermal instability). In the latter case the logarithmic wind distribution cannot be expected to hold within the hollows of the roughness elements, so that it will be necessary in treating evaporation from a hydrodynamically rough sea surface to consider an *intermediate boundary layer* between the turbulent and laminar boundary layers. This fourth division of the whole frictional layer is of small importance for momentum transfer, but is necessary in considering diffusion processes.

[5] For discussions of the extent to which the stress may be considered constant within the turbulent boundary layer see Rossby and Montgomery (1935, pp. 40–44) and Calder (1939).

In the case of a smooth surface Rossby (1936a) has shown that the following general law, developed by von Kármán (1934) for flow over smooth solid boundaries, applies in the turbulent boundary layer:

$$\frac{1}{\gamma} + \frac{1}{k_0} \ln \frac{1}{\gamma} = 5.5 + \frac{1}{k_0} \ln r \tag{7}$$

Here the variable γ is the *coefficient of resistance* defined by

$$\tau_0 = \rho\gamma^2 W^2, \qquad W_* = \gamma W. \tag{8}$$

The other variable has the character of a Reynolds number and is defined by

$$r \equiv \frac{Wz}{\nu}, \tag{9}$$

ν being the kinematic coefficient of molecular viscosity. Equation (7) contains non-dimensional quantities only. It may be written to give the wind speed explicitly as

$$\frac{W}{W_*} = 5.5 + \frac{1}{k_0} \ln \frac{W_* z}{\nu} \cdot \tag{10}$$

The physical significance of von Kármán's law is apparent from the following derivation. Within the laminar layer, whose effective mean thickness is δ,

$$\frac{\tau_0}{\rho} = \nu \frac{dW}{dz}, \tag{11}$$

so that

$$W_\delta = \frac{\tau_0}{\rho} \frac{\delta}{\nu} \cdot \tag{12}$$

Integration of the general relation due to Prandtl and von Kármán for the rate of shear in the turbulent boundary layer (which on integration also gives eq. 2),

$$\frac{dW}{dz} = \frac{W_*}{k_0(z + z_0)}, \tag{13}$$

gives the velocity distribution within this layer,

$$\frac{W - W_\delta}{W_*} = \frac{1}{k_0} \ln \frac{z + z_0}{z_0}. \tag{14}$$

The origin has been placed at the top of the laminar layer. Aside from continuity in velocity involved in this equation, let there also be continuity in shear (which, because stress is constant, involves continuity in viscosity) between the two layers, so that the roughness parameter is prescribed as

$$z_0 = \frac{\nu}{k_0 W_*}, \tag{15}$$

as given by Rossby (1936a, eq. 16). It is to be expected that the thickness of the laminar layer may be described in terms of the viscosity and density of the fluid, and of the stress; the simple dimensional relationship is then

$$\delta = \frac{\lambda\nu}{W_*} \cdot \tag{16}$$

The simultaneous equations (12), (14), (15), (16) solve to yield (10), provided the factor λ is assigned the value 7.8. In finding this solution k_0 is given the value 0.40; furthermore z_0 must be neglected in the sum $z+z_0$, so it is to be expected that von Kármán's law applies only in the region $z \gg z_0$.

If, however, it is assumed that (10) is valid down to the top of the laminar layer, and (10) and (12) are solved for δ, the value $\lambda = 11.5$ results as stated by von Kármán (1934, p. 10) and subsequently by Rossby (1936a, eq. 13) and Sverdrup (1937, p. 6). But this treatment involves a discontinuity in shear at the top of the laminar layer. Sverdrup's (1937, eq. 13 and Fig. 3) empirical value derived from my humidity measurements is $\lambda = 12.4$. But he has used all observations (except those under thermal stability), while it will be found that in many cases the surface is hydrodynamically rough (see p. 26). Hence the determinations from his individual groups are very scattered, and this mean factor cannot be considered decisive.

According to this interpretation of von Kármán's law, then, over a smooth surface the viscosity increases linearly upward from the molecular value at the top of the laminar layer according to

$$\frac{\eta}{\rho} = \nu + k_0 W_* z, \tag{17}$$

which follows from (3).

EVAPORATION FROM A HYDRODYNAMICALLY SMOOTH SEA SURFACE

Within the laminar boundary layer water vapor is transported by molecular diffusion. Hence, denoting evaporation by E (g cm^{-2}sec^{-1}), the kinematic coefficient of diffusion of water vapor through air by κ (cm^{2}sec^{-1}), and specific humidity by q,

$$E = -\kappa\rho\frac{dq}{dz} \cdot \tag{18}$$

If the air in immediate contact with the water is saturated with respect to the water, integration from the sea surface to the top of the laminar layer gives

$$q_s - q_\delta = \frac{E\delta}{\kappa\rho} = \frac{\nu}{\kappa}\,\frac{E\lambda}{\rho W_*} \cdot \tag{19}$$

It may be assumed that eddy diffusivity and eddy viscosity are identical in magnitude (see p. 27) except very near the surface. Both Millar (1937, eq. 7b) and Sverdrup (1937, eqs. 1 and 10) have assumed the identity to hold down to the top of the laminar layer. Since the two molecular coefficients have different magnitudes, this involves a discontinuity in diffusivity at this boundary. In order to maintain continuity it appears better to adopt the following modified form of (17) to express the eddy diffusivity of water vapor within the turbulent boundary layer:

$$\frac{A}{\rho} = \kappa + k_0 W_* z. \tag{20}$$

The vertical transfer of water vapor in the turbulent layer is then

$$E = -A\frac{dq}{dz} = -\rho(\kappa + k_0 W_* z)\frac{dq}{dz}\,. \tag{21}$$

On integration, remembering that the origin is at the top of the laminar layer,

$$q_\delta - q = \frac{E}{\rho k_0 W_*}\ln\frac{\kappa + k_0 W_* z}{\kappa}\,. \tag{22}$$

Except in the lowest few centimeters the first term of the logarithmic expression is negligible. Eliminating q_δ with (19),

$$q_s - q = \frac{E}{\rho k_0 W_*}\left(\frac{\lambda\nu k_0}{\kappa} + \ln\frac{k_0 W_* z}{\kappa}\right). \tag{23}$$

The last equation represents the expected water vapor distribution over a smooth sea surface in terms of the evaporation. It may be written to give the evaporation explicitly in terms of the difference between q_s and the specific humidity at a height b,

$$E = \rho k_0 W_* \frac{q_s - q_b}{\dfrac{\lambda\nu k_0}{\kappa} + \ln\dfrac{k_0 W_* b}{\kappa}}\,. \tag{24}$$

An expression for the evaporation in terms of similar quantities was deduced by Millar (1937, eq. 7g), but his is based in part on his equation 7f, in the present notation $W_\delta = W_*/k_0$. His derivation of this from von Kármán's law (10), assumed applicable down to the top of the laminar layer, contains a mathematical error which invalidates the result.

Eliminating E from the last two equations,

$$q_s - q = (q_s - q_b)\frac{\dfrac{\lambda\nu k_0}{\kappa} + \ln\dfrac{k_0 W_* z}{\kappa}}{\dfrac{\lambda\nu k_0}{\kappa} + \ln\dfrac{k_0 W_* b}{\kappa}}\,. \tag{25}$$

From this it follows that

$$\begin{aligned}\Gamma &\equiv -\frac{1}{e_s - e_b}\frac{de}{d\ln z} = -\frac{1}{q_s - q_b}\frac{dq}{d\ln z}\\ &= \left(\frac{\lambda\nu k_0}{\kappa} + \ln\frac{k_0 W_* b}{\kappa}\right)^{-1} = \left(\frac{\lambda\nu k_0}{\kappa} + \ln\frac{k_0\gamma_a W_a b}{\kappa}\right)^{-1}.\end{aligned} \tag{26}$$

As Millar (1937, p. 53) has pointed out, ν/κ is nearly independent of temperature and may be assigned the fixed value 0.602. Accordingly, for a chosen height b for the standard humidity reading and a chosen anemometer level a, Γ varies only with wind speed and with the temperature effect on γ_a/κ. Using $k_0=0.40$ and $\lambda=7.8$, the last equation may be written

$$\Gamma = \left(0.96 + \ln \frac{\gamma_a W_a b}{\kappa}\right)^{-1}. \tag{27}$$

Rossby's (1936a, Fig. 1) graph of (7) gives γ_a as a function of $r=W_a a/\nu$. Choosing $a=6$ meters and $b=4$ meters, to agree with the present observations, the theoretical values of Γ are as shown in Table 3.

TABLE 3

THEORETICAL Γ-VALUES FOR HYDRODYNAMICALLY SMOOTH SEA SURFACE

T	ν cm²sec⁻¹	κ cm²sec⁻¹	$W_6=1$	$W_6=2$	$W_6=5$	$W_6=10$ m sec⁻¹
0°	0.13	0.22	0.103	0.097	0.089	0.085
20°	0.15	0.25	0.104	0.098	0.090	0.085

EVAPORATION FROM A HYDRODYNAMICALLY ROUGH SEA SURFACE

The theory for the wind distribution above a hydrodynamically rough surface does not tell us the probable distribution of eddy viscosity within the intermediate boundary layer. Above the top of this layer the stress is constant with elevation and equal to the sum of the stress in the laminar boundary layer and of the local pressure gradients against the sides of the waves. On the average this total stress decreases on descending through the intermediate layer and in the lower part reaches its value in the laminar layer. Following Millar (1937, p. 55) it appears simplest as a first approximation to assume that the stress is constant, the friction velocity having a value W_{*K}, and equal to the stress in the laminar layer up to the top of the intermediate layer, and that within this layer conditions obey the von Kármán distribution for a smooth surface as in the preceding section. The eddy viscosity above the intermediate layer is given by (3) in terms of a friction velocity $W_{*R}>W_{*K}$. Thus the shearing stress is constant with elevation except for a discontinuity at the top of the intermediate layer. This involves a discontinuity in rate of shear also, but demanding continuity in velocity will give a relation between W_{*R}/W_{*K} and the thickness of the intermediate layer.

It follows from (23), denoting the thickness of the intermediate boundary layer by Z, that

$$q_s - q_Z = \frac{E}{\rho k_0 W_{*K}}\left(\frac{\lambda \nu k_0}{\kappa} + \ln \frac{k_0 W_{*K} Z}{\kappa}\right). \tag{28}$$

In the turbulent boundary layer eddy viscosity and hence eddy diffusivity are given by (3), so the transport of vapor in this layer is

$$E = -\rho k_0 W_{*R} z \frac{dq}{dz}. \tag{29}$$

Integrating this from Z to z gives

$$q_Z - q = \frac{E}{\rho k_0 W_{*R}} \ln \frac{z}{Z} \cdot \tag{30}$$

Eliminating q_Z by means of (28),

$$q_s - q = \frac{E}{\rho k_0 W_{*R}} \left[\ln \frac{z}{Z} + \frac{W_{*R}}{W_{*K}} \left(\frac{\lambda \nu k_0}{\kappa} + \ln \frac{k_0 W_{*K} Z}{\kappa} \right) \right]. \tag{31}$$

This equation may be re-written in a form corresponding to (24) to give the evaporation explicitly,

$$E = \rho k_0 W_{*R} \frac{q_s - q_b}{\ln \dfrac{b}{Z} + \dfrac{W_{*R}}{W_{*K}} \left(\dfrac{\lambda \nu k_0}{\kappa} + \ln \dfrac{k_0 W_{*K} Z}{\kappa} \right)} \cdot \tag{32}$$

This is equivalent to Millar's (1937, eq. 8e) formula for evaporation from a rough surface except for the bracketed part of the denominator, his being based on the invalid application of von Kármán's law. Finally the result for a rough surface is reached that

$$\begin{aligned} \Gamma &= \left[\ln \frac{b}{Z} + \frac{W_{*R}}{W_{*K}} \left(\frac{\lambda \nu k_0}{\kappa} + \ln \frac{k_0 W_{*K} Z}{\kappa} \right) \right]^{-1} \\ &= \left[\ln \frac{b}{Z} + \frac{W_{*R}}{W_{*K}} \left(0.96 + \ln \frac{W_{*K} Z}{\kappa} \right) \right]^{-1}. \end{aligned} \tag{33}$$

In this formula Z and W_{*R}/W_{*K} are not independent if continuity of velocity is demanded at the top of the intermediate boundary layer. The distribution in the turbulent boundary layer, according to (2), gives

$$\frac{W_Z}{W_{*R}} = \frac{1}{k_0} \ln \frac{Z}{z_0} \cdot \tag{34}$$

The distribution in the intermediate boundary layer, according to (10), gives

$$\frac{W_Z}{W_{*K}} = 5.5 + \frac{1}{k_0} \ln \frac{W_{*K} Z}{\nu} \cdot \tag{35}$$

Solving these two equations,

$$\ln \frac{Z}{z_0} = \frac{5.5 k_0 + \ln \dfrac{W_{*K} z_0}{\nu}}{\dfrac{W_{*R}}{W_{*K}} - 1} \cdot \tag{36}$$

For fixed values of $a = 6$ meters and $b = 4$ meters as before, Γ may be regarded as a function of z_0, W_{*R}/W_{*K} and W_6. The friction velocity W_{*R} is computed from (8) and from

$$\gamma_a = \frac{k_0}{\ln \dfrac{a}{z_0}} \tag{37}$$

(Rossby and Montgomery, 1935, eq. 9), using $k_0 = 0.38$.[6] From the resulting value of W_{*K}, Z may be computed from (36) and then finally Γ. For a number of values of each of the independent parameters the results are presented in Table 4. It will be noticed that no combination of the independent parameters which would lead to a thickness of the intermediate layer exceeding 4 meters is included in the table, for it was assumed in deriving (33) that $Z \leq b$.

Even for the wide range of the independent parameters listed in Table 4, it is found that Γ lies between 0.032 and 0.052 so that it is at least distinctly separated from the theoretical values for Γ in the case of a smooth surface listed in Table 3. Furthermore the range may be reduced by proper choice of z_0 and Z. According to Rossby the roughness parameter may be considered independent of wind speed and equal to 0.6 cm. In regard to the thickness of the intermediate layer, it must be that the full shearing stress $\tau_0 = \rho W_{*R}^2$ is active down to the level of the wave tops. So it appears probable that on the average the intermediate layer extends only to this level, its thickness being approximately half the height from trough to crest. The whole height from trough to crest is roughly

$$\epsilon = \frac{0.3}{g} W_a^2 \tag{38}$$

(Rossby and Montgomery, 1935, p. 94), where g is the acceleration of gravity. Thus, using $z_0 = 0.6$ cm, the *a priori* most likely values of Γ for a rough surface are those listed in Table 5.

THICKNESS OF THE TURBULENT BOUNDARY LAYER

The theoretical thickness of the turbulent boundary layer is

$$H = \frac{3}{\sqrt{2}} \frac{k^2 W_*}{f k_0} \tag{39}$$

(Rossby and Montgomery, 1935, eqs. 22 and 25), where f is the Coriolis parameter, and k is a non-dimensional constant relating to the outer layer of frictional influence and hav-

[6] The value 0.40 for the universal turbulence constant is used in the transformation of (33) and in numerical applications of (36), while in this equation (37) involving only the theory for rough surface I have adhered to Rossby's (1936a, p. 6) use of 0.38. Apparently, however, 0.40 is preferable for rough surface as well as smooth (Rouse, 1938, pp. 242–244).

There is no exact atmospheric corroboration of the value of the universal turbulence constant (cf. p. 4, first sentence). E. H. Taylor (1939) has examined certain data on the flow of water in open channels (depth of order of magnitude of 1 meter). The integral of the surface resistance over the wetted perimeter of a cross-section was known from the measured slope of the free surface. In order for the same surface resistance to result from the observed velocity distribution by using

$$\frac{u_2 - u_1}{u_*} = \frac{1}{k_0} \ln \frac{z_2}{z_1}$$

(which follows from either eq. 7 or eq. 2 for $z \gg z_0$), Taylor found that the universal turbulence constant should have values between 0.22 and 0.43 for the different experimental runs. This suggests that perhaps a different value of the constant obtains in atmospheric phenomena also.

TABLE 4

Theoretical Γ-values for hydrodynamically rough sea surface (at 0°C.)

Assumed			Computed		
z_0 cm	$\frac{W_{*R}}{W_{*K}}$	W_6 m sec^{-1}	W_{*K} cm sec	Z cm	Γ
0.1 (γ_6=0.044)	1.7 2.1 2.3	10 10 10	25.9 21.0 19.1	166 9 4	0.052 0.050 0.049
0.3 (γ_6=0.050)	2.0 2.1 2.3	10 10 10	25.0 23.8 21.7	156 85 33	0.045 0.044 0.043
0.6 (γ_6=0.055)	2.1	10	26.2	346	0.041
	2.2	2 5 10 15	5.0 12.5 25.0 37.5	51 110 196 275	0.051 0.045 0.040 0.038
	2.3	2 5 10 15	4.8 12.0 23.9 35.9	35 71 122 166	0.050 0.044 0.040 0.038
	2.4	2 5 10 15	4.7 11.5 22.9 34.4	26 49 81 108	0.050 0.043 0.039 0.037
	2.6 2.9	10 10	21.2 19.0	42 20	0.038 0.036
1.0 (γ_6=0.059)	2.3 2.6 2.9	10 10 10	25.7 22.7 20.3	317 100 45	0.038 0.036 0.034
2.0 (γ_6=0.067)	2.6 2.9	10 10	25.4 23.1	330 140	0.033 0.032

TABLE 5

Selected theoretical Γ-values for hydrodynamically rough sea surface

W_6, m sec^{-1}	2	5	10	15
ϵ, cm	12	77	306	689
Z, cm	6	38	153	344
Γ	0.046	0.043	0.040	0.039

ing a value of about 0.065. This expression is valid for smooth as well as rough surface, and gives the accompanying values for the two cases if $f=10^{-4}$ (corresponding to latitude 43°).

		1	2	3	4	5	6
W_6, m sec^{-1}		1	2	3	4	5	6
H, m	smooth	8	15	21	28	34	40
	z_0=0.6 cm	12	25	37	49	62	74

In the outer layer of frictional influence the eddy viscosity decreases with height, except when there is sufficient instability to produce active convection. Thus in the outer layer humidity will decrease upward more rapidly than the logarithmic decrease in the turbulent boundary layer.

According to the above values the wind must reach 3 m sec^{-1} with rough surface and exceed 5 m sec^{-1} with smooth surface in order for the boundary layer to extend to 38 meters. For this reason the vapor pressure measured at this level was not used in computing Γ except when the wind exceeded 5 m sec^{-1} (see pp. 11–12).

CHARACTER OF THE OBSERVED VERTICAL VARIATION OF VAPOR PRESSURE

The individual observations do not follow regular curves. The first cause of this irregularity is observational error. The second is the actual fluctuation of vapor pressure at a given height due to turbulence. These fluctuations are of the same order of magnitude as the difference between two levels such that the upper is twice as high as the lower. Furthermore their period is sufficiently long for a psychrometer or hair to follow them with considerable amplitude. Hence the tabulated observations, even though many are the means of several series, must be further smoothed in order to disclose the average vertical variation.

Sverdrup (1937, Fig. 1) has already plotted a number of groups which include most of the observations. Though there are irregularities in the curves, they follow a logarithmic distribution in general. He concludes: "It appears, thus, to be well established that the vapor pressure is a linear function of the logarithm of height, if the potential temperature decreases with height."

There are 42 cases in which observations were made at 1.5 or 2 and at 4, 8 and 38 meters. These are divided into seven groups, for which average values are shown in Table 6 and Figure 3. Two groups are of observations under conditions of thermal stability, none of which were considered by Sverdrup.

TABLE 6

AVERAGE VALUES OF MEASURED VAPOR PRESSURE AT THE VARIOUS LEVELS

GROUP	RANGE OF W_6 m sec^{-1}	AVERAGE W_6 m sec^{-1}	RANGE OF T_S-T_4	AVERAGE T_S-T_4	CASES	e_S mb	e_1 mb	$e_{1.5}$ mb	e_2 mb	e_4 mb	e_8 mb	e_{38} mb	Γ*
A	2.6 to 3.0	2.7	0.7° to 1.7°	1.42°	4	28.22	17.70	—	17.40	17.21	17.19	16.20	0.032
B	2.2 to 2.6	2.5	0.25° to 0.8°	0.46°	4	31.25	—	—	27.52	27.26	27.12	26.60	0.073
C	2.6 to 4.8	4.3	−0.7° to −0.1°	−0.35°	13	27.37	—	24.70	—	24.32	24.19	23.65	0.103
D	4.4 to 4.6	4.6	0.0° to 1.6°	1.06°	7	26.94	—	18.48	—	18.02	17.46	16.61	0.064
E	3.7 to 4.7	4.4	0.45° to 1.35°	1.08°	5	31.96	—	—	20.58	19.74	19.13	17.51	0.086
F	5.9 to 7.1	6.2	0.15° to 0.6°	0.41°	5	30.94	—	—	22.32	21.99	21.25	20.03	0.086
G	5.3 to 6.9	5.9	−0.7° to −0.1°	−0.39°	4	26.88	—	24.46	—	23.91	23.68	23.32	0.162
				total	42								

* Γ is computed from observed e_S, and from e_4 and $de/d \ln z$ according to the straight lines in Figure 3.

For Group B, light wind and slight instability, and for Group C, 4.3 m sec^{-1} and stability, it would be expected according to the last section that the logarithmic distribution would not extend to 38 meters. The figure, however, gives only slight indication of this. Furthermore there is no deviation, consistent among all groups, from the logarithmic distribution.

CARBON DIOXIDE EXCHANGE BETWEEN OCEAN AND ATMOSPHERE

The gases dissolved in sea water undergo an exchange with the atmosphere in the same manner as water vapor. From this point of view Buch has discussed the vertical distribution of carbon dioxide above the water surface on two occasions during *Atlantis* Cruise 45.

The times of the observations are approximately as indicated in Table 1. The air samples, for which Buch (1939, Tabelle 4) gives the carbon dioxide content, were obtained from the mainmast at 30 meters, from the positions of the psychrometers at 8 and 4 meters, and from the trawling platform amidships at one or two lower levels.

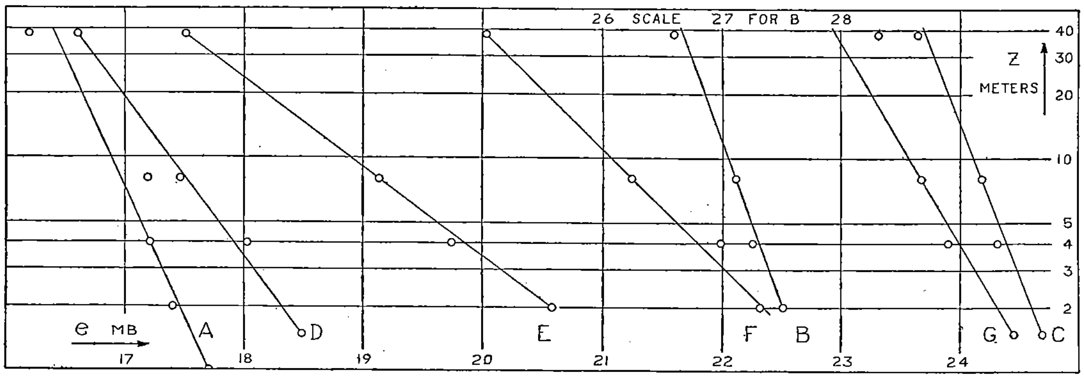

FIG. 3. Vapor pressure (e) in millibars against logarithm of height (z) for the seven groups of Table 6.

Buch (1939, Abb. 2) has plotted the series from July 21, when carbon dioxide increased upward. It follows well a logarithmic distribution up to 8 meters. Above this the increase is more rapid, in accord with the theoretical height—19 meters for smooth surface at this wind speed of 2.6 m sec^{-1} (cf. p. 19)—for the transition from the turbulent boundary layer to the outer layer of frictional influence. From the carbon dioxide content in volume per unit volume of air at NTP, c, at 0.3 and 8 meters it follows that $dc/d \ln z = 0.021 \cdot 10^{-4}$. The mean of the three values of carbon dioxide tension in the surface water (Buch, 1939, Tabelle 3) on this day is $2.83 \cdot 10^{-4}$. At 4 meters the value $3.12 \cdot 10^{-4}$ was observed, so $c_s - c_4 = -0.29 \cdot 10^{-4}$. Thus for carbon dioxide it is found that $\Gamma = 0.07$. This agrees well with the simultaneous determination for water vapor, namely $\Gamma = 0.08$

On July 30 a carbon dioxide content of $3.54 \cdot 10^{-4}$ was found at 20 cm and it decreased upward. The tension of the water measured on this day, but probably not at the same time, was $2.2 \cdot 10^{-4}$. This second series, giving a maximum just above the surface, cannot represent a steady condition in the sense demanded by the present discussion.

MAGNITUDE OF THE OBSERVED VAPOR PRESSURE GRADIENTS

The Γ-values computed theoretically are approximately 0.09 for hydrodynamically smooth surface and 0.04 for hydrodynamically rough surface. In comparing these with the observed values, an important assumption of the theory must be borne in mind, namely that the evaporation takes place only from the actual sea surface. But spray, when present, must contribute a considerable part of the evaporation. The total evaporation is proportional to Γ (provided this is measured above the height to which appreciable spray extends), so that, if for instance spray doubles the rate of evaporation, Γ would also be doubled.

The individual Γ-values in Tables 1 and 2 are quite scattered, and part of this scattering is due to the error of observation. Accordingly mean values have been formed. For this purpose the observations have been divided into three groups according to wind speed (light, intermediate and moderate) and into four groups according to lapse rate (moderate and slight instability, and slight and moderate stability). The latter are defined by the difference between the water temperature at the surface and the air temperature at 4 meters. The results are given in Table 7, which contains all Γ-values from Tables 1 and 2. The grouping of Table 6 was arranged to conform as nearly as possible with that of this table.

TABLE 7

SUMMARY OF Γ-VALUES FROM AUTHOR'S OBSERVATIONS

Range of W_6 m sec^{-1}	Average W_6 m sec^{-1}	Range of T_s-T_4	Average T_s-T_4	Cases	Average Γ	Av. dev. of Γ
2.3 to 3.0	2.6	0.85° to 2.6°	1.60°	13	0.038	0.031
2.2 to 3.0	2.6	0.25° to 0.8°	0.55°	14	0.079	0.029
2.6	2.6	−0.4° to −0.3°	−0.35°	2	0.085	0.005
3.4 to 4.7	4.4	0.9° to 2.5°	1.75°	15	0.058	0.024
3.4 to 5.0	4.2	0.0° to 0.8°	0.39°	14	0.054	0.042
4.3 to 4.9	4.7	−0.4° to −0.9°	−0.25°	11	0.143	0.069
4.6 to 4.7	4.7	−0.7° to −0.45°	−0.60°	3	0.177	0.056
5.4 to 8.4	6.9	0.9° to 2.0°	1.45°	10	0.103	0.016
5.9 to 7.8	6.6	0.15° to 0.8°	0.46°	11	0.134	0.084
—	6.9	—	−0.1°	1	0.05	—
5.2 to 6.9	6.0	−0.9° to −0.6°	−0.72°	7	0.141	0.030
				total 101		

In order to indicate the amount of scattering in the individual Γ-values, the average deviations ($\Sigma|\Gamma-\Gamma_{av}|/N$) are shown in the last column of Table 7. Since these are fairly large, parts of the following discussion must be considered subject to question.

The Γ-value 0.038 for light wind and moderate instability is not far from the theoretical value 0.045 for this wind speed over a rough surface (Table 5). The other two groups for light wind compare well with the theoretical value 0.096 for smooth surface (Table 3). This good agreement for light wind, when there is no spray, is tentatively considered to substantiate the theoretical values, so that discrepancies between observed and theoretical values for intermediate and moderate wind will be ascribed to the effect of spray.

The first two groups for intermediate wind indicate either a mixture of rough and smooth surface or else rough surface with added evaporation from spray (wave crests normally start to break at this wind speed of Beaufort force 3). The second of these, however, appears rather uncertain, for by excluding the five observations on July 30 the average for Γ is increased to 0.084. The latter value would indicate smooth surface for this group at slight instability. The group for slight stability contains the three observations on August 21, when e_s-e_4 was quite small; if these are excluded, the average for Γ is decreased to 0.118. The difference between the latter value and the theoretical one for smooth surface may be due either to spray or to the effect of stability.

Since with moderate wind there is a marked difference between the Γ-values for moderate and for slight instability, it seems best to interpret the first as indicating rough surface and the second as indicating smooth surface, both strongly affected by spray. The group for moderate stability has nearly the same Γ-value as that for indifferent equilibrium, so that stability appears to have only a small effect.

The observations just discussed may be compared with those of Wüst (1937). I have plotted each of his series on a semi-logarithmic scale, omitting the one at the lowest level of 20 cm because it may fall within the intermediate boundary layer, as well as the one at 2 meters made from the ship in his Group II. Γ has been determined from a straight line drawn by eye to agree as closely as possible with the plotted points, and from observed $e_s - e_4$. The observations are divided into the same groups as above, and a summary given in Table 8. The important quantities for the individual observations are listed in Table 9.[7]

TABLE 8

Summary of Γ-values from Wüst's observations

Range of W_6 m sec^{-1}	Average W_6 m sec^{-1}	Range of $T_s - T_4$	Average $T_s - T_4$	Cases	Average Γ	Av. dev. of Γ
0.0 to 2.0	1.3	−1.1° to 0.1°	−0.37°	3	0.107	0.031
4.0	4.0	1.1° to 2.7°	1.88°	4	0.062	0.024
4.0 to 4.3	4.1	0.1° to 0.4°	0.23°	3	0.097	0.044
3.8 to 4.0	3.9	−2.1° to −0.2°	−1.00°	3	0.033	0.016
5.8 to 9.4	6.5	0.9° to 2.1°	1.51°	13	0.065	0.048
6.1 to 8.5	7.1	0.2° to 0.8°	0.50°	5	0.084	0.033
5.5 to 5.8	5.7	−2.1° to −0.5°	−0.95°	5	0.166	0.099
			total	36		

That Tables 7 and 8 agree well on the average shows that there is probably no consistent error in either set of observations.

The group in Table 8 for moderate instability at intermediate wind gives a Γ-value agreeing closely with that of Table 7. The one for slight instability indicates a smooth surface. The very small one for stability appears fictitious.

The group for moderate instability at moderate wind has a Γ-value definitely showing the surface to be rough, but with much less effect of spray than was found in Table 7. The one for slight instability might indicate a mixture of rough and smooth surface. The last group appears noticeably affected by stability.

CHARACTER OF THE SURFACE AND EFFECT OF SPRAY

Simultaneous measurements of vertical wind gradient are given by Wüst (1920, p. 73) for his Group I. These are the only such simultaneous measurements available. Rossby (1936a, p. 9–12) has already discussed the wind measurements. From 1 to 6 meters above the surface the wind speed is proportional to the logarithm of height and indicates a hydrodynamically rough surface with the values of the roughness parameter

[7] For Group I Wüst lists both the Beaufort estimate and the measured wind at 6 meters. One notes that the measured values are much larger in respect to the estimates than given for instance in the Meteorological Office (1936, p. 64–65) scale. Accordingly in Group II, for which only Beaufort estimates are listed, it seemed best to reduce these by means of the accompanying scale based on Group I. Extrapolated values are parenthesized.

Beaufort	0	0–1	1	2	2–3	3	3–4	4
m sec^{-1}	(0.0)	(2.0)	4.0	5.8	6.1	7.8	(8.5)	(9.4)

TABLE 9

THE INDIVIDUAL SERIES OF WÜST'S MEASUREMENTS IN THE BALTIC SEA

		Day and Hour IX, 1939	W_6 m sec^{-1}	T_s	T_s-T_4	e_s-e_4 mb	Γ	z_0* cm
I	1	7,1016	3.7	13.8°	−1.6°	−0.1	—	5.0
	2	1200	4.3	13.6°	−1.8°	−1.2	—	17.0
	3	1352	6.2	13.6°	−1.8°	−0.9	—	1.4
	4	1555	3.8	14.1°	−2.1°	1.2	.04	20.0
	5	1800	5.5	14.2°	−2.1°	0.5	.10	20.0
	6	2010	5.5	14.8°	−0.6°	1.7	.20	4.3
	7	2200	6.3	15.0°	0.2°	2.1	.07	0.8
	8	8,0020	5.8	14.9°	1.0°	2.3	.19	0.6
	9	0200	6.3	14.8°	0.9°	2.3	.02	0.3
	10	0400	6.2	14.7°	1.0°	2.7	−.07	0.025
	11	0618	8.2	14.6°	0.9°	3.3	.13	0.8
	12	0800	7.0	14.1°	0.7°	4.1	.13	0.9
	13	1000	4.3	13.8°	0.2°	3.9	.11	1.6
II	1	14,1200	(0)	18.1°	−1.1°	3.2	.06	
	2	1400	(4.0)	17.6°	0.1°	1.5	.15	
	3	1600	(2.0)	17.3°	0.1°	1.1	.15	
	4	1800	(4.0)	17.3°	0.4°	1.5	.03	
	5	2000	(4.0)	16.8°	1.5°	2.9	.11	
	6	2200	(5.8)	16.5°	1.1°	2.1	.08	
	7	17,0600	(4.0)	15.5°	2.2°	4.0	.03	
	8	0800	(5.8)	15.1°	2.1°	5.2	.05	
	9	1015	(5.8)	15.4°	2.0°	5.1	.02	
	10	1215	(6.1)	15.4°	1.9°	5.9	.04	
	11	1400	(6.1)	14.9°	1.0°	5.6	.06	
	12	1600	(8.5)	15.0°	0.8°	4.7	.04	
	13	18,0600	(5.8)	15.1°	1.6°	3.1	.05	
	14	0800	(6.1)	15.6°	2.0°	4.5	.17	
	15	1000	(7.6)	15.4°	2.0°	3.9	.06	
	16	1230	(4.0)	15.3°	1.1°	3.9	.06	
	17	1400	(6.1)	15.3°	0.4°	3.6	.06	
	18	1600	(5.8)	15.2°	−0.5°	1.7	.38	
	19	1800	(4.0)	15.0°	−0.2°	4.7	.05	
	20	2000	(2.0)	15.1°	−0.1°	3.7	.11	
	21	19,0000	(7.6)	14.6°	0.4°	3.1	.12	
	22	0800	(9.4)	15.3°	2.1°	4.0	.04	
	23	1400	(5.8)	16.3°	−0.9°	3.2	.05	
	24	1600	(5.8)	15.8°	−0.65°	1.7	.10	
	25	1800	(4.0)	15.1°	−0.7°	2.3	.01	
	26	2000	(4.0)	15.5°	2.7°	3.6	.05	

* Reproduced from Rossby (1936a, Table 3).

reproduced in Table 9. These are large in the earlier series, apparently because the wind conditions during series I1–I6 were not stationary (Wüst, 1937, p. 5). The later series give values in the vicinity of $z_0=0.6$ cm. The wind at the 0.2 and 1 meter levels follows closely the von Kármán distribution (7). Thus the wind distribution is in good agreement with the eddy viscosity distribution assumed for a rough surface. The discontinuity in gradient occurs at 1 meter; the average wind for series I7–I13 being 6.3 m sec^{-1} this agrees

TABLE 10

AVERAGE VALUES OF VAPOR PRESSURE IN MILLIBARS FOR FOUR SUB-GROUPS OF WÜST'S GROUP II

Group	Range of W_6 m sec^{-1}	Average W_6 m sec^{-1}	Range of T_s-T_4	Average T_s-T_4	Cases	e_s	From rowboat				From ship				Γ
							$e_{0.2}$	$e_{0.5}$	$e_{1.2}$	e_2	e_2	e_4	e_6	e_9	
II-A	4.0	4.0	1.1° to 2.7°	1.88°	4	17.80	15.13	14.64	14.47	14.33	14.47	14.20	14.16	14.03	0.058
II-B	4.0 to 8.5	6.0	0.1° to 0.8°	0.42°	5	18.05	16.11	15.57	15.44	15.33	15.41	15.20	15.01	14.99	0.069
II-C	4.0 to 5.8	5.1	−0.9° to −0.2°	−0.59°	5	17.49	15.25	14.85	14.83	14.64	14.83	14.48	14.35	14.24	0.087
II-D	5.8 to 9.4	6.5	1.0° to 2.1°	1.76°	9	17.37	14.15	13.60	13.25	13.11	13.31	13.01	12.92	12.67	0.064
				total	23										

with the height of the intermediate boundary layer used in Table 5.

The accompanying mean values of vapor pressure for Wüst's series I7–I13 are plotted

W_6	$T_s - T_4$	e_s	$e_{0.2}$	$e_{0.5}$	$e_{1.2}$	$e_{2.5}$	e_4	e_6
6.3 m sec^{-1}	0.70°	16.45	14.41	14.12	13.88	13.51	13.51	13.48 mb

in Figure 4. The straight line corresponds to $\Gamma = 0.102$. If the eddy viscosity distribution really conforms to that assumed for a rough surface, it must be concluded that in this case Γ and the evaporation are more than doubled by the effect of spray. In the absence of spray a break in the curve would be expected at about 60 cm (Table 5). But it is easily seen that, if the vapor from spray is released uniformly throughout the lowest meter, the break would be much less pronounced.

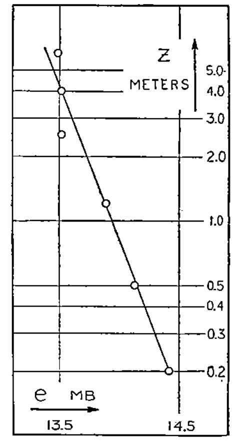

FIG. 4. Vapor pressure against logarithm of height for the last seven observations of Wüst's Group I, when the wind distribution indicated a roughness of $z_0 = 0.6$ cm. The line gives $\Gamma = 0.102$. Scales same as in Figure 3.

The humidity measurements in Wüst's Group II are especially interesting because they are from seven levels between 0.2 and 9 meters, and because he considers them more reliable than Group I. Mean values for each of four sub-groups are shown in Table 10 and in Figure 5. Only the three observations at light wind are omitted.

The average of Γ for the series in Group II-A is 0.062 (see Table 8), indicating a rough surface influenced by spray. The theoretical value for this wind speed is, from Table 5, $\Gamma = 0.044$, and a straight line corresponding to this is drawn in Figure 5. The plotted points suggest a break in slope at 50 cm. This is twice as high as the top of the intermediate boundary layer given in Table 5, but cannot be considered as bad agreement. With $Z = 50$ cm, the theoretical value of Γ is changed only to about 0.0455 (corresponding to $W_{*R}/W_{*K} = 2.35$), as may be seen from Table 4. The expected humidity gradient in the intermediate layer can be computed, assuming that it extends to 50 cm and that $e_{0.5} = 14.54$ mb according to the straight line in Figure 5. The friction velocity in this layer is

$$W_{*K} = \frac{W_{*R}}{2.35} = \frac{0.055 W_6}{2.35} = 9.4 \text{ cm sec}^{-1}.$$

Setting $b = 50$ cm in (27), it follows that in this layer

$$\Gamma_K = \left(0.96 + \ln \frac{W_{*K} b}{\kappa}\right)^{-1} = 0.154.$$

The computed slope is then

$$\frac{de}{d \ln z} = -\Gamma_K(e_s - e_{0.5}) = -0.50 \text{ mb},$$

and this line is drawn in the figure. The agreement between the two straight lines and the observations is good, the somewhat stronger observed gradient being due apparently to a small amount of vapor released from spray below 20 cm.

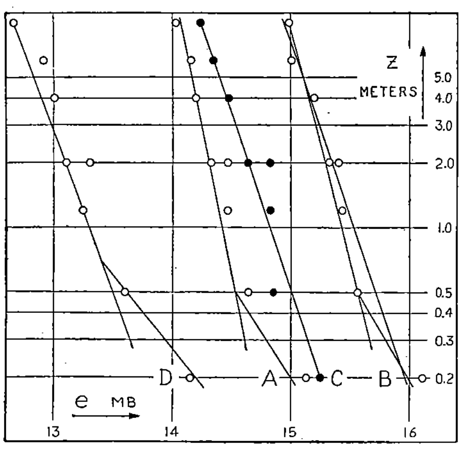

FIG. 5. Vapor pressure against logarithm of height for four sub-divisions of Wüst's Group II, from Table 10. Scales same as in Figures 3 and 4.

The vapor pressures for Group II-A from 0.5 to 9 meters follow extremely closely a straight line corresponding to $\Gamma=0.058$. It should be mentioned that this Γ-value, for $Z=50$ cm, gives $z_0=0.1$ cm according to (33) and (36). So it could be explained by a lower value of the roughness parameter than 0.6 cm, instead of by spray. But Group II-D, for higher wind and with larger Γ, would correspondingly give a still smaller value of the roughness parameter, so this explanation seems unlikely.

The vapor pressures for Group II-D are less regular and in drawing the upper straight line in Figure 5 it seemed best to neglect the ones at 2, 4, 6 meters observed from the ship, for these are less reliable. The line gives $\Gamma=0.064$, indicating that the surface is rough and that spray increases the evaporation by 50%. Assuming the break in humidity gradient to occur at 70 cm, which is this time in agreement with Table 5, it is found in the same manner as above that $\Gamma_K=0.107$. Increasing this by 50% results in the lower straight line. This line is practically as steep as the line joining the observations at 20 and 50 cm, again indicating that the vapor from spray is liberated below 20 cm.

For Group II-B one line is drawn through the points at 0.5 and 9 meters, giving $\Gamma=0.069$. This is 65% higher than the theoretical value for a rough surface. Assuming $Z=50$ cm, it is found that $\Gamma_K=0.113$. This increased by 65% gives the lower line. A single straight line has also been drawn, giving $\Gamma=0.092$. This line for smooth surface does not agree quite so well with the plotted points.

The group shown in Figure 4, for which wind gradients indicated rough surface, falls between Groups II-B and II-D in regard to wind and thermal conditions, but its Γ-value shows much stronger effect of spray. Perhaps I have given too high valuations to the Beaufort estimates for Group II.

Group II-C definitely indicates a smooth surface. The line gives $\Gamma=0.087$, so spray seems to have no effect.

Thus Wüst's observations can be adequately explained by the present theory. The discontinutiy in humidity gradient which he emphasizes at about 50 cm (Wüst, 1937, p. 9) appears to represent the top of the intermediate boundary layer, in other words the average level at which the turbulence associated with the surface waves becomes active.

It is now possible to summarize the results. This is done by reference to the averages of Γ-values from individual series as given in Tables 7 and 8, and to the Γ-values derived from averages of humidity observations as given in Tables 6 and 10.

With moderate instability, that is with $T_s-T_4>0.8°$, the surface is always hydrodynamically rough. Perhaps for light winds the roughness parameter is greater than 0.6 cm.[8] Spray appears to increase Γ and evaporation by about 40% for 4 m sec^{-1}, 100% for 6.5 m sec^{-1}.

With slight instability ($0°\leqq T_s-T_4\leqq 0.8°$) the surface is hydrodynamically smooth for light winds. The different determinations of Γ do not clearly indicate the critical wind speed at which the surface becomes rough, but perhaps it is normally at about 5 m sec^{-1}.

With stability the surface is smooth for winds up to 6 m sec^{-1}, which is the extent of the observations.

The humidity measurements have therefore made it possible to elaborate on Rossby's (1936a, p. 19) conclusions as to the character of the sea surface. Tentative values have been assigned for the two factors, wind speed and stability, which determine whether the surface is rough or smooth under steady conditions over a large surface.

[8] $\Gamma=0.032$ for Group A in Table 6 corresponds to $z_0=1.9$ cm. $\Gamma=0.038$ for the first group in Table 7 corresponds to $z_0=1.2$ cm. Both are computed for 20°C. from (33) and (36), using $W_6=2.6$ m sec^{-1} and $Z=10$ cm.

EFFECT OF THERMAL STABILITY ON EVAPORATION

The theoretical development took no account of stability. Its effect will now be discussed in the light of Sverdrup's investigations over a smooth snow surface.

One question is in regard to the assumed equality of eddy viscosity and eddy diffusivity. Sverdrup (1936a,[9] p. 46) concludes that over the snow surface the two were identical below 7 meters except in his group for light winds and very great stability, where $W_7 = 2.0$ m sec^{-1} and $T_s - T_5 = -4.82°$. In the observations over water made by Wüst and by me $T_s - T_4$ was always less than half this amount. Furthermore the theoretical values for Γ and for evaporation depend only on conditions up to the standard levels a and b, which do not exceed 7 meters. Hence it can be safely concluded that for this purpose eddy viscosity and eddy diffusivity are identical except possibly for winds less than 2 m sec^{-1}, when evaporation is in any case very small.

The other question, in regard to the decrease of eddy viscosity by stability, cannot be answered so definitely. According to Sverdrup conditions up to a limited height are not at all affected by stability. This height for a wind of $W_7 = 5$ m sec^{-1} and for stability corresponding to even $T_s - T_5 = -1.62°$, is nearly 2 meters (Sverdrup, 1936a, Fig. 16b). Over the ocean the height should rarely be less than this except in very light winds. Hence, in comparing adiabatic and stable conditions, it is best to compare two cases in which W_2 and $e_s - e_2$ are the same. Then the stress is the same in the two cases, as well as the distributions below 2 meters of eddy viscosity, wind and vapor, and so are the theoretical values of Γ and evaporation.

Above 2 meters, however, the eddy viscosity in the adiabatic case continues to increase linearly with elevation, while in the stable case, as Sverdrup has shown, it deviates asymptotically from the linear relationship, giving lower values. Likewise the wind in the stable case increases more rapidly with elevation than the logarithmic law, and vapor pressure decreases more rapidly. These two can be well represented above 2 meters by power laws.

Group C in Table 6, which contains more observations than any other stable group, gives $\Gamma = 0.103$. This is only slightly greater than the theoretical value 0.092 for this wind of 4.3 m sec^{-1}, and the difference is easily accounted for by spray. Furthermore it is seen from Figure 3 that the humidity gradient increases only slightly at higher levels. Hence this group can be very little affected by stability. Also in the various other stable groups the difference between the observed Γ-value and the theoretical one for smooth surface appears to be largely due to spray.

The important effect of stability is thus simply to keep the surface smooth. It appears from the observations that the eddy diffusivity in the lowest 8 or 9 meters is otherwise not affected to an important extent for $T_4 - T_s$ up to 1°.

For computing evaporation under conditions of thermal stability it would be preferable to measure wind and humidity at a standard level lower than 6 meters. Then one would be still further assured that the eddy diffusivity is unaffected.

It should be noted that the eddy diffusivity was assumed to be given correctly by (3) even under conditions of instability, when there may be thermal convection. Perhaps this is not permissible.

[9] For a summary of this paper, with additions, see Sverdrup (1938).

SUMMARY AND CONCLUSION

Observations of humidity distribution in the range 1 to 38 meters above the sea surface were tabulated in detail. These follow the logarithmic distribution. It was found

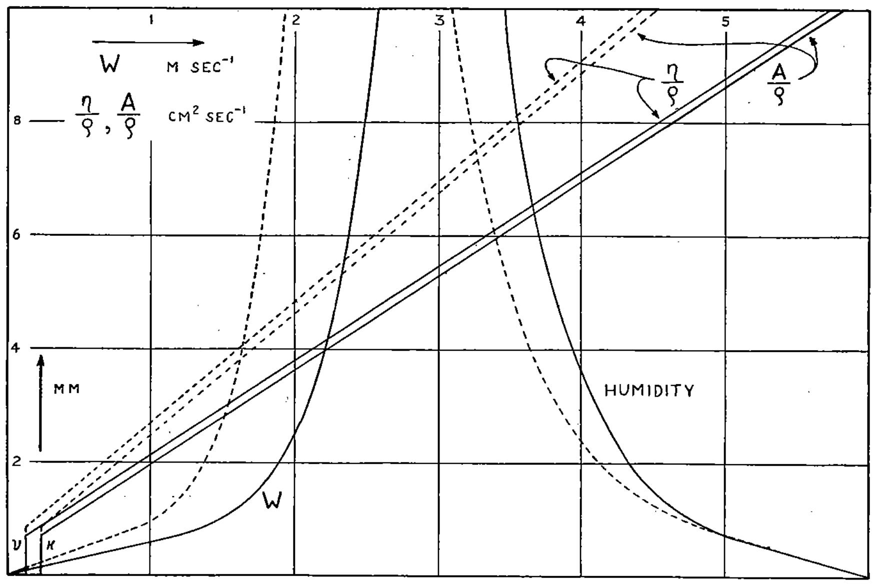

FIG. 6. Assumed distribution of eddy viscosity (η/ρ), eddy diffusivity (A/ρ), wind (W) and humidity in the first centimeter above the water, linear vertical scale. Full line is for a hydrodynamically smooth surface, broken line for a hydrodynamically rough surface. The segment marked ν represents the value of molecular viscosity, that marked κ the value of molecular diffusion of water vapor in air, which obtain in the laminar boundary layer. In both cases $W_6=5$ m sec^{-1} and the humidity is the same at 6 meters. For the case of rough surface a roughness parameter of $z_0=0.6$ cm and a height of the intermediate layer of $Z=38$ cm are used.

convenient to express the vertical humidity gradient for each series according to (6) as the non-dimensional quantity Γ. Although the observations are not the best that could be expected, they were found to agree in many respects with deductions from previous knowledge of the atmospheric layer of frictional influence.

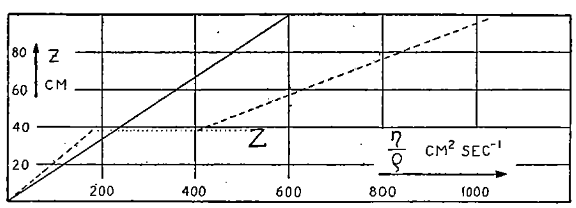

FIG. 7. Distribution of eddy viscosity (η/ρ) in the first meter above the water in the same two cases as in Figure 6, linear vertical scale. The dotted line marked Z represents, for rough surface, the top of the intermediate boundary layer. With this small scale eddy viscosity and eddy diffusivity are indistinguishable.

The humidity gradients were found to depend markedly on the hydrodynamic character of the sea surface and on spray. In turn these two depend not only on wind speed, but also on thermal stability. In fact the transition from hydrodynamically smooth to hydrodynamically rough surface, which has been previously supposed to occur at a critical wind speed, appears from the present study to depend more on stability than on wind. Tentative numerical results concerning these factors are found on page 26.

Figures 6–8 summarize graphically the assumed effect of the hydrodynamic character of the surface, giving a detailed example of the theoretical distributions of eddy viscosity, eddy diffusivity, wind and vapor pressure for both smooth and rough surfaces, neglecting spray.

According to the present development the rate of evaporation is[10]

$$E = \rho k_0 \gamma_a \Gamma (q_s - q_b) W_a. \tag{40}$$

20 m

15 m In this expression ρ is air density, k_0 is the universal turbulence constant, q_s is specific

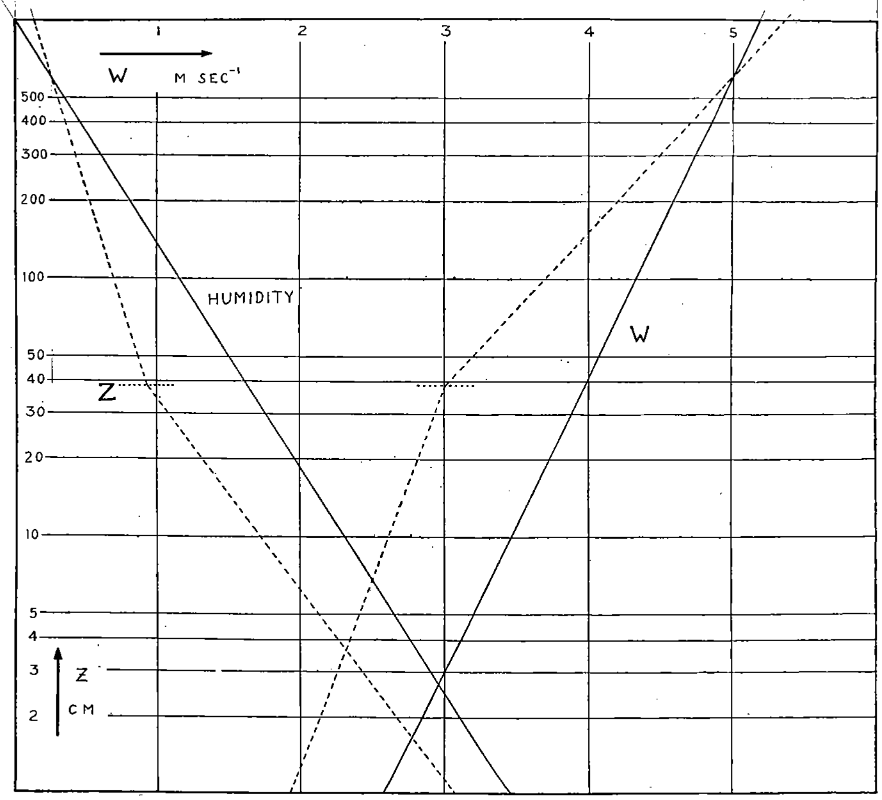

FIG. 8. Distribution of wind (W) and humidity between 1 cm and 10 meters above the water in the same two cases as in Figures 6 and 7, logarithmic vertical scale.

humidity of saturation with respect to the sea surface, q_b is humidity at height b, and W_a is wind speed at height a. Both γ_a, the coefficient of resistance, and Γ, which may suitably be called the *evaporation coefficient*, depend on the hydrodynamic character of the surface. The dependence is such, however, that their product is about the same for smooth as for rough surface. If theoretical values for the evaporation coefficient according to (26) or (33) are used, the equation gives only the evaporation that would occur in the absence of spray. But if one uses observed values of the evaporation coefficient from heights above the layer where vapor is released from spray, the equation gives the total evaporation.

[10] This follows by combining (24) and (26) or (32) and (33), using (8).

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